Maths IA – Maths Exploration Topics

Important

Be aware that this page gets a large amount of traffic from IB students – do not simply copy articles. This will almost certainly be spotted by the IB moderators and could result in you failing your diploma. Use this resource like you would a good wiki – as a starting point and inspiration for your own personal investigation.

Before choosing a topic you need to read this page which gives very important guidance from the IB. Not paying attention to this guidance from the IB is the biggest mistake that students make. It could easily mean the difference between coursework which gets 17/20 and one which gets 11/20. That will probably cost you at least 1 IB grade. Do **not** skip this step!

Algebra and number

1) Modular arithmetic – This technique is used throughout Number Theory. For example, Mod 3 means the remainder when dividing by 3.

2) Goldbach's conjecture: "Every even number greater than 2 can be expressed as the sum of two primes." One of the great unsolved problems in mathematics.

3) Probabilistic number theory

4) Applications of complex numbers: The stunning graphics of Mandelbrot and Julia Sets are generated by complex numbers.

5) Diophantine equations: These are polynomials which have integer solutions. Fermat's Last Theorem is one of the most famous such equations.

6) Continued fractions: These are fractions which continue to infinity. The great Indian mathematician Ramanujan discovered some amazing examples of these.

7) Patterns in Pascal's triangle: There are a large number of patterns to discover – including the Fibonacci sequence.

8) Finding prime numbers: The search for prime numbers and the twin prime conjecture are some of the most important problems in mathematics. There is a $1 million prize for solving the Riemann Hypothesis and $250,000 available for anyone who discovers a new, really big prime number.

9) Random numbers

10) Pythagorean triples: A great introduction into number theory – investigating the solutions of Pythagoras' Theorem which are integers (eg. 3,4,5 triangle).

11) Mersenne primes: These are primes that can be written as $2^n - 1$.

12) Magic squares and cubes: Investigate magic tricks that use mathematics. Why do magic squares work?

13) Loci and complex numbers

14) Egyptian fractions: Egyptian fractions can only have a numerator of 1 – which leads to some interesting patterns. 2/3 could be written as 1/6 + 1/2. Can all fractions with a numerator of 2 be written as 2 Egyptian fractions?

15) Complex numbers and transformations

16) Euler's identity: An equation that has been voted the most beautiful equation of all time, Euler's identity links together 5 of the most important numbers in mathematics.

17) Chinese remainder theorem. This is a puzzle that was posed over 1500 years ago by a Chinese mathematician. It involves understanding the modulo operation.

18) Fermat's last theorem: A problem that puzzled mathematicians for centuries – and one that has only recently been solved.

19) Natural logarithms of complex numbers

20) Twin primes problem: The question as to whether there are patterns in the primes has fascinated mathematicians for centuries. The twin prime conjecture states that there are infinitely many consecutive primes (eg. 5 and 7 are consecutive primes). There has been a recent breakthrough in this problem.

21) Hypercomplex numbers

22) Diophantine application: Cole numbers

23) Perfect Numbers: Perfect numbers are the sum of their factors (apart from the last factor). ie 6 is a perfect number because 1 + 2 + 3 = 6.

24) Euclidean algorithm for GCF

25) Palindrome numbers: Palindrome numbers are the same backwards as forwards.

26) Fermat's little theorem: If p is a prime number then $a^p - a$ is a multiple of p.

27) Prime number sieves

28) Recurrence expressions for phi (golden ratio): Phi appears with remarkable consistency in nature and appears to shape our understanding of beauty and symmetry.

29) The Riemann Hypothesis – one of the greatest unsolved problems in mathematics – worth $1million to anyone who solves it (not for the faint hearted!)

30) Time travel to the future: Investigate how traveling close to the speed of light allows people to travel "forward" in time relative to someone on Earth. Why does the twin paradox work?

31) Graham's Number – a number so big that thinking about it could literally collapse your brain into a black hole.

32) RSA code – the most important code in the world? How all our digital communications are kept safe through the properties of primes.

33) The Chinese Remainder Theorem: This is a method developed by a Chinese mathematician Sun Zi over 1500 years ago to solve a numerical puzzle. An interesting insight into the mathematical field of Number Theory.

34) Cesaro Summation: Does $1 – 1 + 1 – 1 … = 1/2?$. A post which looks at the maths behind this particularly troublesome series.

35) Fermat's Theorem on the sum of 2 squares – An example of how to use mathematical proof to solve problems in number theory.

36) Can we prove that $1 + 2 + 3 + 4 …. = -1/12$? How strange things happen when we start to manipulate divergent series.

37) Mathematical proof and paradox – a good opportunity to explore some methods of proof and to show how logical errors occur.

38) Friendly numbers, Solitary numbers, perfect numbers. Investigate what makes a number happy or sad, or sociable! Can you find the loop of infinite sadness?

39) Zeno's Paradox – Achilles and the Tortoise – A look at the classic paradox from ancient Greece – the philosopher "proved" a runner could never catch a tortoise – no matter how fast he ran.

40) Stellar Numbers – This is an excellent example of a pattern sequence investigation. Choose your own pattern investigation for the exploration.

41) Arithmetic number puzzle – It could be interesting to do an exploration where you solve number problems – like this one.

Geometry

1a) Non-Euclidean geometries: This allows us to "break" the rules of conventional geometry – for example, angles in a triangle no longer add up to 180 degrees. In some geometries triangles add up to more than 180 degrees, in others less than 180 degrees.

1b) The shape of the universe – non-Euclidean Geometry is at the heart of Einstein's theories on General Relativity and essential to understanding the shape and behavior of the universe.

2) Hexaflexagons: These are origami style shapes that through folding can reveal extra faces.

3) Minimal surfaces and soap bubbles: Soap bubbles assume the minimum possible surface area to contain a given volume.

4) Tesseract – a 4D cube: How we can use maths to imagine higher dimensions.

5) Stacking cannon balls: An investigation into the patterns formed from stacking canon balls in different ways.

6) Mandelbrot set and fractal shapes: Explore the world of infinitely generated pictures and fractional dimensions.

7) Sierpinksi triangle: a fractal design that continues forever.

8) Squaring the circle: This is a puzzle from ancient times – which was to find out whether a square could be created that had the same area as a given circle. It is now used as a saying to represent something impossible.

9) Polyominoes: These are shapes made from squares. The challenge is to see how many different shapes can be made with a given number of squares – and how can they fit together?

10) Tangrams: Investigate how many different ways different size shapes can be fitted together.

11) Understanding the fourth dimension: How we can use mathematics to imagine (and test for) extra dimensions.

12) The Riemann Sphere – an exploration of some non-Euclidean geometry. Straight lines are not straight, parallel lines meet and angles in a triangle don't add up to 180 degrees.

13) Graphically understanding complex roots – have you ever wondered what the complex root of a quadratic actually means graphically? Find out!

14) Circular inversion – what does it mean to reflect in a circle? A great introduction to some of the ideas behind non-euclidean geometry.

15) Julia Sets and Mandelbrot Sets – We can use complex numbers to create beautiful patterns of infinitely repeating fractals. Find out how!

16) Graphing polygons investigation. Can we find a function that plots a square? Are there functions which plot any polygons? Use computer graphing to investigate.

17) Graphing Stewie from Family Guy. How to use graphic software to make art from equations.

18) Hyperbolic geometry – how we can map the infinite hyperbolic plane onto the unit circle, and how this inspired the art of Escher.

19) Elliptical Curves– how this class of curves have importance in solving Fermat's Last Theorem and in cryptography.

20) The Coastline Paradox – how we can measure the lengths of coastlines, and uses the idea of fractals to arrive at fractional dimensions.

21) Projective geometry – the development of geometric proofs based on points at infinity.

Calculus/analysis and functions

1) The harmonic series: Investigate the relationship between fractions and music, or investigate whether this series converges.

2) Torus – solid of revolution: A torus is a donut shape which introduces some interesting topological ideas.

3) Projectile motion: Studying the motion of projectiles like cannon balls is an essential part of the mathematics of war. You can also model everything from Angry Birds to stunt bike jumping. A good use of your calculus skills.

4) Why e is base of natural logarithm function: A chance to investigate the amazing number e.

5) Fourier Transforms – the most important tool in mathematics? Fourier transforms have an essential part to play in modern life – and are one of the keys to understanding the world around us. This mathematical equation has been described as the most important in all of physics. Find out more! (This topic is only suitable for IB HL students).

6) Batman and Superman maths – how to use Wolfram Alpha to plot graphs of the Batman and Superman logo

7) Explore the Si(x) function – a special function in calculus that can't be integrated into an elementary function.

Statistics and modelling

1) Traffic flow: How maths can model traffic on the roads.

2) Logistic function and constrained growth

3) Benford's Law – using statistics to catch criminals by making use of a surprising distribution.

4) Bad maths in court – how a misuse of statistics in the courtroom can lead to devastating miscarriages of justice.

5) The mathematics of cons – how con artists use pyramid schemes to get rich quick.

6) Impact Earth – what would happen if an asteroid or meteorite hit the Earth?

7) Black Swan events – how usefully can mathematics predict small probability high impact events?

8) Modelling happiness – how understanding utility value can make you happier.

9) Does finger length predict mathematical ability? Investigate the surprising correlation between finger ratios and all sorts of abilities and traits.

10) Modelling epidemics/spread of a virus

11) The Monty Hall problem – this video will show why statistics often lead you to unintuitive results.

12) Monte Carlo simulations

13) Lotteries

14) Bayes' theorem: How understanding probability is essential to our legal system.

15) Birthday paradox: The birthday paradox shows how intuitive ideas on probability can often be wrong. How many people need to be in a room for it to be at least 50% likely that two people will share the same birthday? Find out!

16) Are we living in a computer simulation? Look at the Bayesian logic behind the argument that we are living in a computer simulation.

17) Does sacking a football manager affect results? A chance to look at some statistics with surprising results.

18) Which times tables do students find most difficult? A good example of how to conduct a statistical investigation in mathematics.

19) Introduction to Modelling. This is a fantastic 70 page booklet explaining different modelling methods from Moody's Mega Maths Challenge.

20) Modelling infectious diseases – how we can use mathematics to predict how diseases like measles will spread through a population

21) Using Chi Squared to crack codes – Chi squared can be used to crack Vigenere codes which for hundreds of years were thought to be unbreakable. Unleash your inner spy!

22) Modelling Zombies – How do zombies spread? What is your best way of surviving the zombie apocalypse?

Surprisingly maths can help!

23) Modelling music with sine waves – how we can understand different notes by sine waves of different frequencies. Listen to the sounds that different sine waves make.

24) Are you psychic? Use the binomial distribution to test your ESP abilities.

25) Reaction times – are you above or below average? Model your data using a normal distribution.

26) Modelling volcanoes – look at how the Poisson distribution can predict volcanic eruptions, and perhaps explore some more advanced statistical tests.

27) Could Trump win the next election? How the normal distribution is used to predict elections.

28) How to avoid a Troll – an example of a problem solving based investigation

29) The Gini Coefficient – How to model economic inequality

30) Maths of Global Warming – Modeling Climate Change – Using Desmos to model the change in atmospheric Carbon Dioxide.

31) Modelling radioactive decay – the mathematics behind radioactivity decay, used extensively in science.

Games and game theory

1) The prisoner's dilemma: The use of game theory in psychology and economics.

2) Sudoku

3) Gambler's fallacy: A good chance to investigate misconceptions in probability and probabilities in gambling. Why does the house always win?

4) Bluffing in Poker: How probability and game theory can be used to explore the the best strategies for bluffing in poker.

5) Knight's tour in chess: This chess puzzle asks how many moves a knight must make to visit all squares on a chess board.

6) Billiards and snooker

7) Zero sum games

8) How to "Solve" Noughts and Crossess (Tic Tac Toe) – using game theory. This topics provides a fascinating introduction to both combinatorial Game Theory and Group Theory.

9) Maths and football – Do managerial sackings really lead to an improvement in results? We can analyse the data to find out. Also look at the finances behind Premier league teams

10) Is there a correlation between Premier League wages and league position? Also look at how the Championship compares to the Premier League.

11) The One Time Pad – an uncrackable code? Explore the maths behind code making and breaking.

12) How to win at Rock Paper Scissors. Look at some of the maths (and psychology behind winning this game.

13) The Watson Selection Task – a puzzle which tests logical reasoning. Are maths students better than history students?

Topology and networks

1) Knots

2) Steiner problem

3) Chinese postman problem – This is a problem from graph theory – how can a postman deliver letters to every house on his streets in the shortest time possible?

4) Travelling salesman problem

5) Königsberg bridge problem: The use of networks to solve problems. This particular problem was solved by Euler.

6) Handshake problem: With n people in a room, how many handshakes are required so that everyone shakes hands with everyone else?

7) Möbius strip: An amazing shape which is a loop with only 1 side and 1 edge.

8) Klein bottle

9) Logic and sets

10) Codes and ciphers: ISBN codes and credit card codes are just some examples of how codes are essential to modern life. Maths can be used to both make these codes and break them.

11) Zeno's paradox of Achilles and the tortoise: How can a running Achilles ever catch the tortoise if in the time taken to halve the distance, the tortoise has moved yet further away?

12) Four colour map theorem – a puzzle that requires that a map can be coloured in so that every neighbouring country is in a different colour. What is the minimum number of colours needed for any map?

13) Telephone Numbers – these are numbers with special properties which grow very large very quickly. This topic links to graph theory.

14) The Poincare Conjecture and Grigori Perelman – Learn about the reclusive Russian mathematician who turned down $1 million for solving one of the world's most difficult maths problems.

Mathematics and Physics

1) The Monkey and the Hunter – How to Shoot a Monkey – Using Newtonian mathematics to decide where to aim when shooting a monkey in a tree.

2) How to Design a Parachute – looking at the physics behind parachute design to ensure a safe landing!

3) Galileo: Throwing cannonballs off The Leaning Tower of Pisa – Recreating Galileo's classic experiment, and using maths to understand the surprising result.

4) Rocket Science and Lagrange Points – how clever mathematics is used to keep satellites in just the right place.

5) Fourier Transforms – the most important tool in mathematics? – An essential component of JPEG, DNA analysis, WIFI signals, MRI scans, guitar amps – find out about the maths behind these essential technologies.

6) Bullet projectile motion experiment – using Tracker software to model the motion of a bullet.

7) Quantum Mechanics – a statistical universe? Look at the inherent probabilistic nature of the universe with some quantum mechanics.

Further ideas:

1) Radiocarbon dating – understanding radioactive decay allows scientists and historians to accurately work out something's age – whether it be from thousands or even millions of years ago.

2) Gravity, orbits and escape velocity – Escape velocity is the speed required to break free from a body's gravitational pull. Essential knowledge for future astronauts.

3) Mathematical methods in economics – maths is essential in both business and economics – explore some economics based maths problems.

4) Genetics – Look at the mathematics behind genetic inheritance and natural selection.

5) Elliptical orbits – Planets and comets have elliptical orbits as they are influenced by the gravitational pull of other bodies in space. Investigate some rocket science!

6) Logarithmic scales – Decibel, Richter, etc. are examples of log scales – investigate how these scales are used and what they mean.

7) Fibonacci sequence and spirals in nature – There are lots of examples of the Fibonacci sequence in real life – from pine cones to petals to modelling populations and the stock market.

8) Change in a person's BMI over time – There are lots of examples of BMI stats investigations online – see if you can think of an interesting twist.

9) Designing bridges – Mathematics is essential for engineers such as bridge builders – investigate how to design structures that carry weight without collapse.

10) Mathematical card tricks – investigate some maths magic.
Voting systems

11) Flatland by Edwin Abbott – This famous book helps understand how to imagine extra dimension. You can watch a short video on it here

12) Towers of Hanoi puzzle – This famous puzzle requires logic and patience. Can you find the pattern behind it?

13) Different number systems – Learn how to add, subtract, multiply and divide in Binary. Investigate how binary is used – link to codes and computing.

14) Methods for solving differential equations – Differential equations are amazingly powerful at modelling real life –

from population growth to to pendulum motion. Investigate how to solve them.

15) Modelling epidemics/spread of a virus – what is the mathematics behind understanding how epidemics occur? Look at how infectious Ebola really is.

16) Hyperbolic functions – These are linked to the normal trigonometric functions but with notable differences. They are useful for modelling more complex shapes.

17) Medical data mining – Explore the use and misuse of statistics in medicine and science.